How to use this Teacher's Guide

The teacher's guide is divided into three main sections: Introduction, Lesson Plans and Glossary of Words.

INTRODUCTION

This introduction section covers the following topics that the teachers should be aware of:

1. Overview of Student Centred Pedagogy
2. Basic Principles for Effective Teaching and Learning
3. Blooms Taxonomy
4. Questions that could be asked at different stages of a lesson
5. Overview of Classroom Level Student Assessment
6. Overview of 21st Century Skills and Soft Skills
7. Overview of Social Dimensions in Secondary School Classrooms (Gender, ethnic, social status and disabilities)
8. Syllabus and Year Plan

1. Overviewing of Student-Centred Pedagogy

In a student-centred secondary school classroom the following key points will be observable:

- Lessons are interesting, relevant and meaningful to student's lives
- Student participate actively and in collaboration with their peers
- Teachers provide challenge and real-life problem-solving situations
- Students develop higher level critical thinking and problem-solving situations
- Teacher explains, asks questions and listens; students discuss, ask questions and listen

2. Basic Principles of Effective Teaching and Learning

Effective Teaching and Learning occurs when teachers and students work together to achieve learning outcomes as follows:

- The teacher makes learning clear for the learner → $\boxed{\text{content and knowledge}}$
- The teacher and students use a variety of approaches → $\boxed{\text{pedagogy}}$
- The teacher and students give and receive clear and regular feedback → $\boxed{\text{assessment}}$

Learning Outcomes are statements that clearly define what students are expected to know, understand and be able to do in terms of knowledge, attitudes and values at the end of a period of learning (see Lesson Plans)

3. Bloom's Taxonomy

Bloom's Taxonomy of Learning Domains are incorporated in lesson plans and activities to ensure that students in secondary school grades develop higher-order thinking skills. Look for these key words in lesson plans.

Bloom's Taxonomy

4. Questions that could be asked at different stages of a lesson

Below is a list of questions the teacher can ask at different stages of a lesson. At the Beginning of a Lesson:

- What skills and knowledge will the students learn by the end of this topic / lesson?

- How will the students be organized for the planned activities?
- What information do you need? What information do the students need?
- What equipment or materials do you need? What equipment or materials do the students need?
- How will the materials be available, organized and used?
- What do you think the result will be?

In the Middle of a Lesson:

- Ask yourself "Is the lesson/activity going as expected?"
- Is there any other information you might need to help the students understand the lesson?

At the End of a Lesson:

- Did your work/the activity turn out as expected?
- Did the lesson raise other questions for the students to consider?
- Could it have been done differently?
- What have you / the students learned in this lesson?
- Has the work helped you to assess individual student's conceptual understanding?
- What needs to be included in the next lesson or improved in this lesson?

5. Overview of 21st Century Skills and Soft Skills

Young people need 21st century skills and soft skills in the workplace and to navigate our complex world. The 5 C's - important skills for learning are:

- Collaboration - encourage students to work in groups, share ideas, and find solutions together

- Communication - verbal and non-verbal communication: reading, writing, speaking and listening
- Critical thinking and problem solving - let students find solutions to problems and correct errors
- Creativity and innovation - thinking "outside the box" to explore new ideas and solve problems
- Citizenship - active participation in the school community, fairness and conflict resolution skills

6. Overview of Classroom Level Assessment

90% of all assessment for learning happens in classrooms during lessons. Classroom level assessment can be used to provide immediate feedback to students and teachers. It helps to identify what learning the students need to do next if they are to successfully move forward with their learning. Teachers can make observations during a lesson to identify instructional strategies that work.

(1) Classroom Level assessment includes the following approaches:

- **Student-centered**: teachers and students focus on observing and improving learning rather than teaching. Students understand what they are being asked to learn (not what they are asked to do).
- **Teacher-directed**: each individual teacher decides what to assess, how to assess and how to respond to the information gained through the assessment.
- **Active participation of students**: by being part of the assessment process their understanding and self-assessment skills increases. Their motivation is also increased when they realize that the teacher is interested in them as learners.
- **Formative assessment**: The intention of formative assessment is to inform the teacher about a student's learning progress by providing evidence, so the teacher can help the student improve their learning.

Teachers will need to determine how best they can record this evidence from the students learning so they will be able to adjust their teaching to improve the quality of learning. Development of the soft skills is an ideal place to begin to develop the teacher's formative assessment capability and techniques. Every teacher is a teacher of the soft skills, so all students are having their learning reinforced and enhanced in every lesson.

(2) Strategies for conducting classroom level formative assessments

- **Observation**: Observing the students directly and recording observations, including behavior in group work in preparation for practical or performance activities. Also observe one or more of the 5Cs.
- **Questioning**: The Teacher asks the students questions to determine the level of understanding (Blooms Taxonomy) and adjusts their teaching as a result. This may happen at any time in the lesson as well in the review and assessment for the final stage of a lesson or series of lessons.
- **Student Learning Journal**: The teacher asks students to write answers to open ended questions (e.g. what I have learned and what I still need to learn) in their exercise book as part of the review at the end of the lesson, and then at the beginning of the next lesson a similar task is used to determine prior knowledge, by asking students to write what they know about today's lesson objective. The teacher uses a sample of maybe 10 books/students to assess the overall class performance or prior knowledge and adjust teaching accordingly.

7. Overview of Social Dimensions in Secondary School Classrooms (gender, ethnic, social status and disabilities)

The National Education Law (2014) states that the curriculum should produce good citizens who understand and accept diversity, value equality, value democratic and human rights standards, allow promotion of each ethnic group's rich literature, culture, arts, traditions and historical heritage, and which is also in line with the curriculum used in other countries.

As a basic principle, the secondary school classroom should be “inclusive” of all students, regardless of their gender, ethnic, social status and disability, An equitable and inclusive school environment will enable all students to access education while respecting individual identity and values, and gaining an understanding and appreciation of the diversity of others.

Teachers are required to demonstrate actions as good examples of desired positive attitudes and values. Those attitudes and values will inspire students to become good citizens.

Look for other examples and opportunities that strengthen equity and inclusiveness in the classroom setting, in lessons and school environment. You can also discuss and share ideas with other teachers in your school to ensure all students are valued and respected.

Chapter 1

Introduction to Coordinate Geometry

Total number of lesson periods (15), 1 period (45 minutes)

Learning Outcomes
It is expected that students will be able to

- know idea of locations of points in four quadrants
- specify the points and lines in Rectangular Cartesian coordinate system
- find the length of a line segment and the coordinates of midpoint
- evaluate the slope of a line
- study the algebraic equation on graph
- understand how to draw the graph of linear equations
- know how to write the linear equation by using two forms (slope-intercept form and point-slope form)

Skill Development
It is expected that the students will be able to

- solve problems on direct application of formulas

- learn about analytical methods of studying the properties of geometric figures based upon what they have studied in middle school
- construct a table for ordered pairs and get idea of plotting graphs on XY-plane
- sketch the graph of the lines in coordinate plane
- analyze some properties of figures through applications
- apply their skills to solve geometry problems
- solve applications of linear equations

Section 1.1 Midpoint and Length of a Line Segment

Number of lesson periods (4)

Lesson Objectives

After this lesson, students will be able to:

- apply midpoint formula to find endpoints and midpoint of a line segment.
- use the Pythagoras Theorem to find the length of the hypotenuse.
- find distance between two points in a coordinate plane.

Introduction

The midpoint formula is applied when you need to find the center point, halfway between two given points. The distance formula is based on the Pythagoras Theorem. You will learn how to use the Pythagoras Theorem to find the distance between two coordinate points. The Midpoint Formula and the Distance Formula are useful in geometry situations where we want to find the point halfway between two points and the distance between two points.

Suggestion for teaching, practicing and evaluation

Teacher should explain the coordinates of endpoints of a line segment can help students to find the midpoint and the length of a line segment. In this section students will review Pythagoras Theorem to find the distance of two points of the hypotenuse. When calculating the hypotenuse (the length of a line segment), find the difference in the two x values and the difference between the two y values. Students will practice these skills in their independent work. Some examples and practice questions are shown in the text. To help the students more understanding, ask the questions through a real situation.

Section 1.2 Slope of a Straight Line

Number of lesson periods (5)

Lesson Objectives

The students will be able to:

- understand the idea of slope, define the slope formula, learn a brief discussion of special cases of zero slope and undefined slope.
- calculate the slope of a line given two points.
- find the slope of a line from its graph.
- find the slope of the lines $x = a$ and $y = b$.

Introduction

This lesson will introduce the concept of slope to students. Students will need to be given a simple ideas of what is meant by a slope. Teacher should explain slope is calculated as the ratio of the value of vertical change to horizontal change. The easiest way to remember is **rise** over **run**. Students will find it easier to relate simple diagrams or examples in the class room or in their surrounding. Students will need to understand that the slope of the line passing through two points on a graph (x_1, y_1) and (x_2, y_2) is given by $\dfrac{y_2 - y_1}{x_2 - x_1}$ and that $y_2 - y_1$ is the vertical difference and $x_2 - x_1$ is the horizontal difference between the two points. This lesson would be incorporated in a unit on graphing linear equations.

Suggestion for teaching, practicing and evaluation

Teacher should explain clearly the slope of a line could be positive, negative, zero or undefined. In the case of positive slope, y increases as x increases, so the line slopes upwards to the right. In the case of negative slope, y decreases as x increases, so the line slopes downwards to the right. If y does not change

as x increases, it gives a horizontal line, the slope of any horizontal line is always zero. Be careful when you are explaining slope of a vertical line. In particular, the concept of slope simply does not work for vertical lines. It does not have a defined slope.
For example, let us consider the slope of two points $A(3,1)$ and $B(3,6)$.
The slope of $AB = \frac{6-1}{3-3} = \frac{5}{0} =$ undefined.
Therefore a vertical line has no slope.
At the end of the lesson teacher should make oral test or additional exercises to check for student understanding. Then student will have encourage to determine what they need to be ready to solve the higher level problems.
For example:

- Explain the nature of a line by means of slope.
- If three points are collinear, what can you tell about these slopes?

Section 1.3 Lines in the Coordinate Plane

Number of lesson periods (6)

Lesson Objectives

The students will be able to

- find the slope and intercepts from the equation of a line.
- use the slope-intercept form to identify the slope and y-intercept of a line.
- solve for the slopes of parallel and perpendicular lines.
- determine the equation of a straight line using the slope-intercept form and the point-slope form.
- write an equation of a vertical and horizontal lines.
- graph special forms of equations of lines.
- understand how to find and draw the equation of a line.

Introduction

The slope-Intercept form of a linear equation is written as $y = mx + c$, where m is the slope and c is the value of y at the y-intercept, which can be written as $(0, c)$. It may help you to write the equation of a line if you know the slope and y-intercept of that line. The slope-intercept form can also help you know the slope and a point on the line or two points on the line.
When lines in the coordinate plane are parallel, they have the same slope.
For example, two lines $y = 2x + 3$ and $y = 2x - 7$ are parallel because they have a slope of 2.
When lines are perpendicular, their slopes are negative reciprocals of each other, except for vertical lines causing its slope to be undefined.
For example, two lines $y = -\frac{3}{2}x + 1$ and $y = \frac{2}{3}x - 7$ are perpendicular because these slopes $-\frac{3}{2}$ and $\frac{2}{3}$ are negative reciprocals.
To graph linear equations, you may use

- A table of values.
- The x-intercept and y-intercept.
- The slope-intercept method.

Suggestion for teaching, practicing and evaluation

In this section teacher should explain the followings:

- How to write an equation for a horizontal line and vertical line.
- How to write the equation of a line in slope-intercept form if slope and one point are given.
- How to write the equation of a line in slope-intercept form if two points are given.
- How to write the equation of a line in slope-intercept form if one point and parallel line are given.
- How to write the equation of a line in slope-intercept form if one point and perpendicular line are given.

At the end of the lesson teacher should make oral test or additional exercises to check for student understanding. Then student will have encourage to determine what they need to be ready to solve the higher level problems.
For example:

- What can you say about the slope of two lines which are parallel and which are perpendicular?
- If three vertices of a triangle are given, what is your calculation procedure to prove a given triangle is a right triangle?

SUMMARY

- If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two distinct points on the line,

$$\text{Midpoint of } AB = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{Length of } AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1))^2}$$

$$\text{Slope of } AB = \frac{y_2 - y_1}{x_2 - x_1}$$

Definition and Properties of Slope

- The slope of a line is the ratio of change in y-coordinates to change in x-coordinate (rise over run). Slope is represented by the letter m.
- The followings are the different ways of expressing of slope:

$$\text{Slope } m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y coordinates}}{\text{change in x coordinates}} = \frac{y_2 - y_1}{x_2 - x_1}$$

- A line that increase from left to right has positive slope.
- A line that decrease from left to right has negative slope.

- A horizontal line has zero slope.
- A vertical line has no slope.
- Parallel lines have same slope.
- Perpendicular lines have slopes that are negative reciprocals of each other. The product of their slopes is -1.

Equations of Lines

- **Equation of a Horizontal line**: cutting the y-axis at $(0, a)$ is $y = a$, where a is a constant.
- **Equation of a Vertical line**: cutting the x-axis at $(b, 0)$ is $x = b$, where b is a constant.
- **Slope-Intercept Form**: $y = mx + c$, where m is the slope and c is y-intercept.
- **Point-Slope Form**: $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.

Chapter 2

Exponents and Radicals

Total number of lesson periods (15), 1 period (45 minutes)

Learning Outcomes
It is expected that students will

- compute the powers with integral exponents and rational exponents
- apply the laws of exponents to simplify expressions involving rational exponents
- write exponential expressions in radical form and radical expressions in exponential form
- add, subtract, multiply and divide the radical expressions
- rationalize monomial and binomial denominators in radical expressions
- solve the exponential equations.

Skill Development
It is expected that the students will

- have gained the ability to solve the problems with exponents and radicals.
- have gained critical skill for those who move on to higher level mathematics classes

- easily evaluate the algebraic calculations

Section 2.1 Exponents

Number of lesson periods (7)

Lesson Objectives
The students will be able to

- define key terms in relation to exponents.
- simplify an algebraic expression with integral exponents and rational exponents.
- apply the properties of exponents including multiplication and division of powers with the same base.
- understand that exponents with different bases can't be multiplied.
- find the principal n^{th} roots.
- use the laws of exponents to simplify expressions with rational exponents.

Introduction
In this section, we will introduce the key terms of exponent. If a is any real number and n is a positive integer, then the n^{th} power of a is

$$\underbrace{a \times a \times a \times a \times \cdots \times a}_{n\ factors} = a^n$$

where the number a is called the base and n is called the exponent or index. Then, we define the positive integral exponents, the zero exponents and negative integral exponents. Next we present the rules for integral exponents. Now we will extend the exponential concept to fractional exponents.

Suggestion for teaching, practicing and evaluation
In discussing power involving positive integral exponents, there was no restriction on the base. In extending our definition of power to include zero and negative exponents, we required the base to be different from zero. The extension to rational exponents involves the further restriction of the base. The following facts need emphasis:

- If n is a positive integer and x and y are real numbers, such that $x^n = y$, then x is called the n^{th} root of y.

- If n is odd, there is only one real n^{th} root of y, no matter whether y is negative or zero or positive. In this case the real n^{th} root of y is denoted by $\sqrt[n]{y}$ and is called **the principal n^{th} root of y**.

  For example,
  the principle 5^{th} root of $243 = \sqrt[5]{243} = 3$ and
  the principle 5^{th} root of $-243 = \sqrt[5]{-243} = -3$.

- If n is even and y is positive, there are two real n^{th} roots of y, one positive and the other negative. In that case the positive n^{th} root of y is denoted by $\sqrt[n]{y}$ and is called **the principal n^{th} root of y**.
  For example, the principle fourth root of $625 = \sqrt[4]{625} = 5$.

- For a real number x and an integer n $(n \geq 2)$, $x^{\frac{1}{n}} = \sqrt[n]{x}$, when n is even, x must be positive or zero.

- If m and n are positive integers and $\dfrac{m}{n}$ is a rational number in lowest terms, then for any real number x,
  $$x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m,$$
  when n is even, x must be positive or zero.

- It is important to recognize the difference between expressions such as $(-2)^4$ and -2^4.

Section 2.2 Radicals

Number of lesson periods (3)

Lesson Objectives
The students will be able to

- write exponential expressions in radical form and radical expressions in exponential form.

- apply the rules for radicals.

- change the expressions to the same radical and simplify the radical expressions to simplest form.
- rationalizing the denominators with a monomial containing radical.
- demonstrate an understanding of exponents and radicals as they apply to problems involving integral exponents, rational exponents and radicals.

Introduction
In this section, we introduce the key terms of radical. Then we define the rules for radicals.

Suggestion for teaching, practicing and evaluation
To illustrate the relationship between the radical and exponential form, students may need practice in recognizing these form of equivalence.
The following facts need emphasis:

- $(\sqrt[n]{x})^n = \sqrt[n]{x^n} = x$. Student should understand that this fact follows form the rules for radicals.

Section 2.3 Operations with Radicals

Number of lesson periods (3)

Lesson Objectives
The students will be able to

- add, subtract, multiply and divide the radicals.
- calculate the simplified solution by rationalizing the denominators.

Introduction
In this section, we introduce how to add, subtract, multiply and divide the radicals. When working with radicals, which of the radicals can be added (or subtracted) and which of the radicals can be multiplied (or divided).

Suggestion for teaching, practicing and evaluation.
Solving the problems involving radicals, the following facts need emphasis:

- Two radicals of the same order can be multiplied by using **Rule 2** of radicals. Two radicals of different orders can be multiplied after changing to the equivalent radicals of the same order.

- The radicals with the same index and same radicand are called similar radicals and they can be added or subtracted.

- The sum or difference of dissimilar radicals cannot be simplified.

- Do not confuse the expression $\sqrt{x} + \sqrt{y}$ with the expression $\sqrt{x+y}$.

- When rationalizing the denominator with a monomial containing radical, if the radical in the denominator is a square root, then you multiply by a square root that will give you a perfect square under the radical when multiplied by the denominator. If the radical in the denominator is a cube root, then you multiply by a cube root that will give you a perfect cube under the radical when multiplied by the denominator and so forth...

- When rationalizing a denominator with two terms, you multiply by its conjugate. This process is based on the fact that $(a+b)(a-b) = a^2 - b^2$.

Section 2.4 Exponential Equations

Number of lesson periods (2)

Lesson Objectives

- The students will be able to solve the exponential equations.

Introduction

In this section, we will look at solving the exponential equations. Some exponential equations may be solved using the fact that, if $x^n = x^m$, then $n = m$ for $x \neq 0$ and $x \neq 1$.

Suggestion for teaching, practicing and evaluation

If $x^n = x^m$, then $n = m$ for $x \neq 0$ and $x \neq 1$. We will notice that this fact does require that the base in both exponents to be the same.

SUMMARY

Important Definitions and Rules

For any real numbers b, x, y and positive integers m, n,

1. $b^n = b \cdot b \cdot b \cdots b$ (n factor)
2. $b^0 = 1$ $(b \neq 0)$
3. $b^{-n} = \frac{1}{b^n}$, $\frac{1}{b^{-n}} = b^n$ $(b \neq 0)$
4. $If x^n = y$, then x is the n^{th} root of y.
5. $x^{\frac{1}{n}} = \sqrt[n]{x}$, when n is even, x must be positive or zero.
6. $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$, when n is even, x must be positive or zero.
7. for any real number $x \neq 0$, $x^{-\frac{m}{n}} = \frac{1}{x^{\frac{m}{n}}}$.

Rules for Exponents

For any real numbers x, y and for any rational numbers m, n,

Rule 1 $x^m \cdot x^n = x^{m+n}$

Rule 2 $\frac{x^m}{x^n} = \begin{cases} x^{m-n}, & \text{if} \quad m > n \\ 1, & \text{if} \quad m = n \\ \frac{1}{x^{n-m}}, & \text{if} \quad m < n, x \neq 0 \end{cases}$

Rule 3 $(x^m)^n = x^{mn}$

Rule 4 $(xy)^n = x^n \cdot y^n$

Rule 5 $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$, $y \neq 0$

Rules for Radicals

For any real numbers x, y and for any positive integers m, n and k,

Rule 1 $(\sqrt[n]{x})^n = \sqrt[n]{x^n} = x$

Rule 2 $\sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{xy}$

Rule 3 $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$

Rule 4 $\sqrt[m]{\dfrac{x}{y}} = \dfrac{\sqrt[m]{x}}{\sqrt[m]{y}}, \qquad y \neq 0$

Rule 5 (a) $\sqrt[n]{x^m} = \sqrt[kn]{x^{km}}$
(b) $\sqrt[n]{x^m} = \sqrt[\frac{n}{k}]{x^{\frac{m}{k}}}, \qquad k \neq 0$

Key for Exercises
base, power, index, exponent, root, radical

Chapter 3

Logarithms

Total number of lesson periods (10), 1 period (45 minutes)

Learning Outcomes

The learners are able to

- understand the definition and properties of logarithms.
- convert the logarithmic equations to the corresponding exponential ones, and vice versa.
- use logarithms as a tool to solve some problems, in which certain exponential expressions may involved.
- understand the two specific logarithms – the common logarithm and the natural logarithm – and use them to solve some equations relating to real-life problems.

Skill Development

- Students will be familiar with scientific notation and be able to read and write expressions in these notation.
- Arithmetic operations on decimal numbers can be performed in a reasonable precision.

- Equations in exponential or in logarithmic form can be switched to and from each other so that the equations can be solved easily.
- Properties of logarithms are going to be recognized in comparison with those of exponents.
- Logarithms can be computed by changing their bases whenever necessary.
- Students will be able to use the well-known logarithms in practical ways.

Section 3.1 Scientific Notation

Number of lesson periods (2)

Lesson Objective

Scientific notation and the rounding rules for the results in arithmetic operations will have to be taught.

Introduction

Teachers will have introduced their students with some scientific constants or measurements, which are very small or very large, in ordinary form and let them to read aloud. Then write these numbers in scientific notation beside the original ones, and let the students to read aloud. Ask them which of the two forms is easier to read and write, and say that these are in fact pairwise equivalent.

Suggestion for teaching, practicing and evaluation

Immediately after the introduction, students will have been explained the structure of scientific notation, including the significant digits concerned. Then teachers will have to discuss their students about Examples 1 and 2. After that write a number with different decimal precision on the board and ask what is the difference between the expressions and what does it mean. The answer is the accuracy, which in arithmetic operation obey the general rule stated in the text. The rule should be explained, followed by the group discussion on Examples 3 and 4.

Final part is Exercise 3.1, which can be executed as a guided homework or class work. But brief explanation will be needed for the students to perform their tasks easier. To evaluate the students' skill, teachers can produce additional problems similar to those of examples and exercises and arrange tests, but computations should be easy enough to perform mentally. Scientific calculators may be helpful for the teachers to generate such extra problems as well as to check up the answers.

Section 3.2 Definition of the Logarithm

Number of lesson periods (2)

Lesson Objectives

Definition of logarithm will have to be taught in relation with exponential expressions or equations. The students should not forget the fact that a logarithm can be considered as a kind of exponent.

Introduction

The introduction will be the discussion on the solutions of some exponential equations such as $2^x = 8$, along with the basic property of real numbers.

Suggestion for teaching, practicing and evaluation

The introduction must directly be followed by the definition of logarithm, and then by the explanation of the two equivalent forms of numbers with the help of the diagram shown in text. After considering the consequences, discuss on examples 5 through 9 and problems in Exercise 3.2 in appropriate order. Practicing and evaluation should be done in a similar manner as in section 3.1.

Section 3.3 Properties of Logarithms

Number of lesson periods (3)

Lesson Objectives

Properties of logarithms will have to be discussed. Compare these properties with those of exponential in a relevant manner.

Introduction

Referring to the second line of the first paragraph of this chapter, to multiply two positive numbers means to add the logarithms of these numbers. Similar result holds for the division of a positive number by another positive number.

Suggestion for teaching, practicing and evaluation

The introduction leads to the three properties of logarithms in Theorem 1. Discussion on the proofs is important in order to develop reasoning and thinking skills. Summarize the properties of logarithm by comparing with those of exponents. Now we go on examples 10 through 14 and problems in Exercise 3.3 in appropriate order. Practicing and evaluation should be done in a similar manner as in section 3.1.

Section 3.4 Change of Base

Number of lesson periods (2)

Lesson Objective

A specific rule to change the base of a logarithm will have to be taught.

Introduction

Before going to the general statement of Theorem 2, lets have a check on it for a particular case. Using the fact that $\log_2 3 = 1.5850$, $\log_{10} 3 = 0.4771$, $\log_{10} 2 = 0.3010$, we can check that $\dfrac{\log_{10} 3}{\log_{10} 2} = \dfrac{0.4771}{0.3010} = 1.5850 = \log_2 3$, this illustrates the result in Theorem 2.

Suggestion for teaching, practicing and evaluation

Now state and prove the result in Theorem 2. Then continue to those of Corollaries 1, 2, 3 successively. Finally move on to examples 15 through 18 and problems in Exercise 3.4 in appropriate order. Practicing and evaluation should be done in a similar as in section 3.1.

Section 3.5 Common Logarithm an Natural Logarithm

Number of lesson period (1)

Lesson Objective

Narrow down the study of logarithms to the cases of particular bases 10 and e, where e is an irrational number known as Euler's number.

Introduction

The common logarithm can be introduced by discussion on the background knowledge for the term "pH", which can usually be seen on water bottles. This indeed is a logarithmic scale on the concentration of hydrogen ions in a solution. Richter scale and decibel are another logarithmic scales. Refer to the internet or some other books and give extra knowledge for motivation.

Suggestion for teaching, practicing and evaluation

After inducing common logarithms, explain the characteristic and mantissa with respect to it. Example 19 is about those numbers. Example 20 illustrate an application of natural logarithm to solve an exponential equation, which deals with continuous compounding formula or a special type of compound interest. In Example 21, logarithm behave a tool to evaluate an numerical expression involving multiplication and division. Examples and problems in Exercise 3.5 can be discussed in appropriate order. Practicing and evaluation should be done in a similar manner as in section 3.1.

SUMMARY

1. Scientific Notation:

A positive number is *in scientific notation* when it is written in the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer. The figures or digits in a are called *significant figures.*

In addition and subtraction, the result should be rounded so that it has *the same number of digits after the decimal* as the measurement with the least number of digits to the right of the decimal.

In multiplication and division, the result should have *the same number of significant figures* as the measurement with the least.

2. Definition of the Logarithm:

$\log_b N = n$ if and only if $b^n = N$

Consequently, $b^{\log_b N} = N$, $\log_b b^x = x$, $\log_b 1 = 0$, $\log_b b = 1$.

3. Properties of Logarithm:

$$\log_b (MN) = \log_b M + \log_b N$$

$$\log_b M^k = k \log_b M$$

$$\log_b \left(\tfrac{M}{N}\right) = \log_b M - \log_b N$$

4. Change of Base:

$$\log_a N = \frac{\log_b N}{\log_b a}$$

$$\log_a N = \frac{1}{\log_N a}$$

$$\log_{b^n} N = \frac{1}{n} \log_b N$$

$$a^{\log_k b} = b^{\log_k a}$$

5. Common Logarithm and Natural Logarithm:

$\log_{10} N$ is known as a *common logarithm* and is usually written as $\log N$.

$\log_e N$ is known as a *natural logarithm* and is usually written as $\ln N$, where e is an irrational number (approximately equal to 2.71828).

Chapter 4

Functions

Total number of lesson periods (15), 1 period (45 minutes)

Learning Outcomes
It is expected that students will

- describe the product set from given sets
- distinguish between the concept of relations and functions
- demonstrate the basic function including linear, quadratic, absolute value, square root and rational function
- illustrate and interpret these functions by graphs
- use the graph of function to determine whether the given function is one-to-one or not
- determine equality of functions and one-to-one function,
- evaluate the inverse function and composite function

Skill Development
It is expected that the students will

- analyze problems, apply concepts and make a plan of approaching way to problem

- express the mathematical idea and process in different ways
- comprehend mathematics in environment

Section 4.1 Product sets

Number of lesson periods (1)

Lesson Objectives
At the end of the lesson the students will be able to define the product sets for any sets.

Introduction
Let a and b be any two elements. When we indicate such two elements a and b as an ordered, we take a as the first element and b as the second element and enclose the elements in parentheses (a, b). A pair of such elements a and b, written as (a, b), is called **an ordered pair**. Furthermore we can count the number of elements of $A \times B$. In general, if A has m elements and B has n elements, then $A \times B$ has mn elements.

Suggestion for teaching, practicing and evaluation
The order of elements in the ordered pair is important, for instance $(2, 1)$ and $(1, 2)$ are two different ordered pairs.

Section 4.2 Relations

Number of lesson periods (3)

Lesson Objectives
At the end of the lesson the students will be able to understand the concept of relation.

Introduction
A **relation** is a set of ordered pairs. For example, $R = \{(2, 1), (4, 2), (6, 3)\}$ is a relation. If R is a relation, the set of all first elements x of ordered pairs $(x, y) \in R$ is called the **domain** of R, denoted by $\text{dom}(R)$. The set of second elements y is called the **range** of R, denoted by $\text{ran}(R)$.

Suggestion for teaching, practicing and evaluation

Section 4.3 Functions

Number of lesson periods (11)

At the end of the lesson the students will be able to

- distinguish the concept between relations and functions.
- find the domain of a function
- determine whether two functions are equal or not.

Introduction

Function is an important foundation in mathematics. Most of the real world problems are formulated by mathematical models which are constructed by functions. For instance, when he/she is driving a bike or a car, his/her location is a function of time. Therefore teachers should explain the concepts of function.

Suggestion for teaching, practicing and evaluation

You and your students should consider the following questions:

(1) What are the differences between relation and function?
(2) Is every function a relation? (or) Is every relation a function?
(3) How do the students check above case by vertical line test?
(4) What is domain of function? How do the students find it?

Subsection 4.3.1 The graph of a function

Lesson Objectives

At the end of the lesson the students will be able to draw the basic functions.

Introduction

Let f be any function. The graph of f is the set of ordered pairs given by the function f as

$$\{(x, y) \mid x \in \text{dom}(f) \text{ and } y = f(x)\}.$$

The graph of function f is illustrated by plotting the set of ordered pairs in xy-plane.

The students will study the following curves of basic functions.

Linear function: $y = mx + c$ where m and c are constants.
Quadratic function: $y = ax^2$ where a is a constant.
Absolute valued function: $y = |x|$.
Square root function: $y = \sqrt{x}$.
Rational function: $y = \dfrac{ax+b}{cx+d}$ where a, b, c and d are constants.

Suggestion for teaching, practicing and evaluation

It is enough the student understand the shape of function. Drawing graph of function will be studied in chapter 5 and 6.

Subsection 4.3.2 One-to-one function

Lesson Objectives

At the end of the lesson the students will be able to check whether the given function is one-to-one or not.

Introduction

A function f is called one-to-one

$$\text{if } x, y \in \mathrm{dom}(f), \text{ and } x \neq y \text{ implies } f(x) \neq f(y).$$

In other words

$$\text{if } x, y \in \mathrm{dom}(f), \text{ and } f(x) = f(y), \text{ then } x = y.$$

It is easy to check whether the given function is one-to-one or not by using the horizontal-line test.

> **Horizontal line test:** A real valued function is **one-to-one** if every horizontal line intersects the graph of the function at most one point.

Suggestion for teaching, practicing and evaluation

Teacher should explain the connection of one-to-one definition and horizontal line test. Moreover the important point for this section is finding domain of function. Here is a question, why domain of function is important?

Subsection 4.3.3 Inverse function

Lesson Objectives

At the end of the lesson the students will be able to find the inverse function.

Suggestion for teaching, practicing and evaluation

Teacher should explain some function has inverse but some are not. Why? Here **domain of function** is important again. We can restrict the domain of function to be one-to-one, then we can find its inverse. See example 16.

Subsection 4.3.4 Composition of function

Lesson Objectives

At the end of the lesson the students will be able to

- understand the concept of composition of functions.
- find the domain of composite function.
- evaluate the formula of composite function.

Introduction

If f and g are functions such that $\mathrm{ran}(f) \subset \mathrm{dom}(g)$, then the **composite function** of f and g is the new function $g \circ f$ (read as g **circle** f) with $\mathrm{dom}(g \circ f) = \mathrm{dom}(f)$ such that

$$(g \circ f)(x) = g(f(x))$$

for all $x \in \mathrm{dom}(f)$.

Suggestion for teaching, practicing and evaluation

The most important fact for this subsection is how to find the domain of composition of functions. Teacher should care when the functions are fractional.

SUMMARY

- **Product Sets:**Let A and B be any sets.

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

- **Function Notation:** $(x, y) \in f; \quad f : x \mapsto y; \quad y = f(x)$.
- **Vertical line test:** A relation is a function if every vertical line intersects the graph at most one point.
- **One-to-one function:** A function f is called one-to-one

$$\text{if } x, y \in \text{dom}(f), \text{ and } x \neq y \text{ implies } f(x) \neq f(y)$$

  In other words

$$\text{if } x, y \in \text{dom}(f), \text{ and } f(x) = f(y), \text{ then } x = y.$$

- **Horizontal line test:** If every horizontal line intersects the graph of function at most one point, such function is called one-to-one function.
- **Inverse Function:** Let D be the domain and R be the range of a function f. Suppose that f is a one-to-one function. The inverse function f^{-1} is defined by

$$x = f^{-1}(y) \Leftrightarrow y = f(x)$$

- **Composite of f and g:** Let dom(f) and dom(g) be domain of f and g respectively. The composite of f and g is given by

$$(f \circ g)(x) = f(g(x)),$$

  where Domain of $f \circ g = \{x \mid x \in \text{dom}(g) \text{ and } g(x) \in \text{dom}(f)\}$.

  Closure Property: Linear functions are closed under composition. One-to-one functions are closed under composition.

  Associative Property: Let $f : A \to B$, $g : B \to C$ and $h : C \to D$. Then

$$h \circ (g \circ f) = (h \circ g) \circ f.$$

Identity Function: Let $f : \mathbb{R} \to \mathbb{R}$ and $I : \mathbb{R} \to \mathbb{R}$ be the identity function on $\mathbb{R}$. Then

$$f \circ I = I \circ f = f.$$

Chapter 5

Quadratic Functions

Total number of lesson periods (10), 1 period (45 minutes)

Learning Outcomes
Students will learn

- quadratic functions
- quadratic inequality

Skill Development
Student will learn

- graph of the quadratic functions
- discriminant of quadratic functions
- solving quadratic equations and quadratic inequality

Section 5.1 Graph of the function $y = x^2 + bx + c$

Number of lesson periods (1)

Lesson Objective

- Draw graph of the function $y = x^2 + bx + c$.

Introduction
This lesson may be first step to draw a quadratic function $y = x^2 + bx + c$. By using graph paper (grid paper) students are not difficult to plot the points and draw the functions.

Suggestion for teaching, practicing and evaluation
First draw the functions $y = x^2$ and $y = -x^2$. See by these graphs students will know what a parabola open up is and what a parabola open down is. Then only emphasizes on $y = x^2$. On same graph paper draw the functions such as $y = x^2$, $y = x^2 + 3$ and $y = x^2 - 4$. From these graphs students will learn vertical translations like up and down of the graphs of the functions. After that on same graph paper draw the functions such as $y = x^2$, $y = (x-2)^2$, and $y = (x+3)^2 + 3$. From these graphs students will learn horizontal translations the graphs of the functions. See the figure below for teaching aids. After that try to draw the graphs of the functions as in Example 1. Some more graphs are on the teaching aids.

Changing $y = x^2 + bx + c$ to the form $y = (x - h)^2 + k$ is a necessary part to understand quadratic functions. Start the teaching with easily changeable functions such as $y = x^2 + 2x + 2$, $y = x^2 - 2x + 5$.

Section 5.2 Graph of the function $y = -x^2 + bx + c$

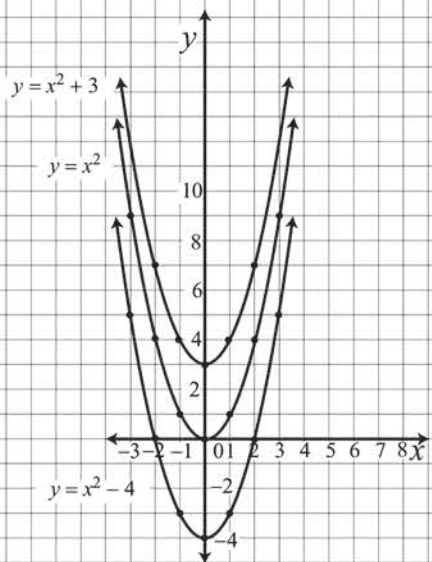
Number of lesson period (1)

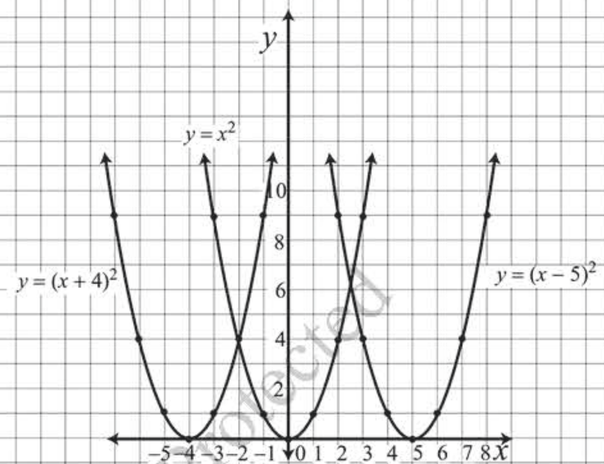
Lesson Objective

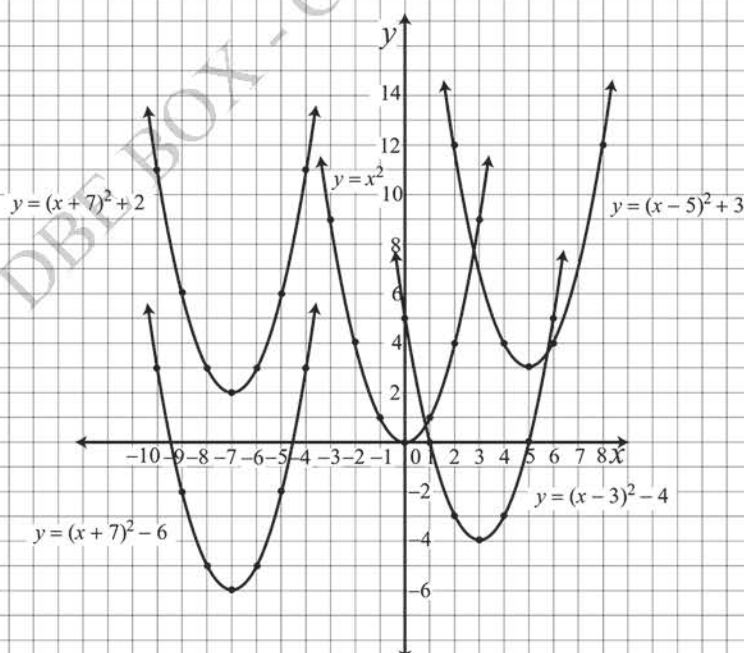
- Draw graph of the functions $y = -x^2 + bx + c$.

Introduction
This lesson may be first step to draw a quadratic function $y = -x^2 + bx + c$. By using graph paper (grid paper) students are not difficult to plot the points and draw the functions.

Suggestion for teaching, practicing and evaluation
First only emphasize on $y = -x^2$. Then continue by using same teaching procedure as in previous section. On same graph paper draw the functions such as $y = -x^2$, $y = -x^2 + 3$ and $y = -x^2 - 4$. From these graphs students will learn vertical translations like up and down of the graphs of the functions.


$y = x^2 + 3$
$y = x^2$
$y = x^2 - 4$


$y = x^2$
$y = (x + 4)^2$
$y = (x - 5)^2$


$y = (x + 7)^2 + 2$
$y = x^2$
$y = (x - 5)^2 + 3$
$y = (x - 3)^2 - 4$
$y = (x + 7)^2 - 6$

After that try to draw the graphs of the functions as in Example 2. Make some teaching aids like the given as in previous section.

As in previous section, changing $y = -x^2 + bx + c$ to the form $y = -(x-h)^2 + k$ is a necessary part to understand quadratic functions. Start the teaching with easily changeable functions such as $y = -x^2 + 2x + 2$, $y = -x^2 - 2x + 5$.

Section 5.3 Graph of the function $y = ax^2$

Number of lesson period (1)

Lesson Objective

- Draw graph of the functions $y = ax^2$.

Introduction

Drawing the quadratic function $y = ax^2$ when $a \neq 0$ is a necessary to know the general quadratic functions.

Suggestion for teaching, practicing and evaluation

Draw at least two graphs for $a > 0$ and for $a < 0$ as shown in the text. If possible add some more drawing. Students need to see the vertical stretches of the graphs.

Section 5.4 Graph of the function $y = ax^2 + bx + c$

Number of lesson periods (2)

Lesson Objectives

- Draw graph of the functions $y = ax^2+bx+c$ and solve maximum/minimum problems related to quadratic functions.

Introduction

This section is the main part of the chapter. Drawing general quadratic function $y = ax^2 + bx + c$ and changing standard form to vertex form is important to understand the nature of quadratic functions. Introduce also here that solving maximum/minimum problems by using quadratic functions.

Suggestion for teaching, practicing and evaluation
May be needed some teaching aids like in section 5.1.

Section 5.5 Discriminant of a Quadratic Function

Number of lesson period (1)

Lesson Objectives

- Find the discriminant of $y = ax^2 + bx + c$ and find relation between discriminant and graph of $y = ax^2 + bx + c$.

Introduction
Discriminant $b^2 - 4ac$ can be calculated for any quadratic functions. When $b^2 - 4ac > 0$ the graph passes through the x-axis at two points. When $b^2 - 4ac < 0$ the graph does not pass through the x-axis. When $b^2 - 4ac = 0$ the graph meets the x-axis at exactly one point.

Suggestion for teaching, practicing and evaluation
Vertex form $y = a(x + \frac{b}{2})^2 - \frac{b^2-4ac}{4a}$ can be used to tell when the values of the quadratic function are positive, zero or negative.

Section 5.6 Quadratic Formula of $ax^2 + bx + c = 0$

Number of lesson period (1)

Lesson Objectives

- Find quadratic formula and use it to solve the quadratic equation $ax^2 + bx + c = 0$.

Introduction
Quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is calculated.

Suggestion for teaching, practicing and evaluation
Formula derivation of quadratic equation $ax^2 + bx + c = 0$ needs to explain.

Section 5.7 Miscellaneous Exercises

Number of lesson period (1)

Lesson Objectives

- Solve miscellaneous exercises.

Introduction

Problem solving by using quadratic equations is introduced in this section.

Suggestion for teaching, practicing and evaluation

There are many problems for this section. Courage students to find some exercises and solve by themselves.

Section 5.8 Quadratic Inequality

Number of lesson periods (2)

Lesson Objective

- Solve the quadratic inequalities.

Introduction

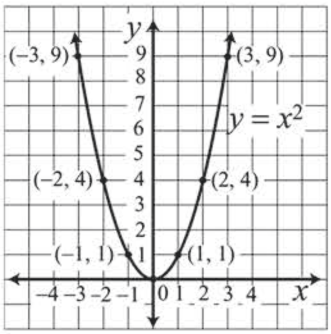
Every quadratic inequality can be solved. This section explains a method for solving the quadratic inequality.

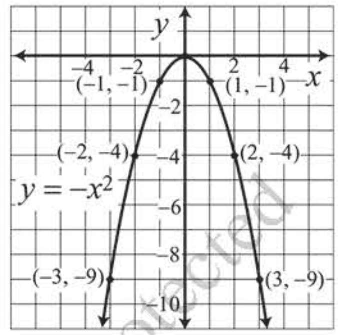
Suggestion for teaching, practicing and evaluation

Six cases of quadratic functions have to teach. After that there is no difficulty for solving quadratic inequality.

SUMMARY

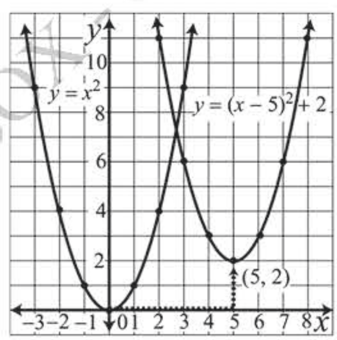
Standard Form: A function $y = ax^2 + bx + c$ where $a \neq 0$ is a **quadratic function** in the **standard form**.





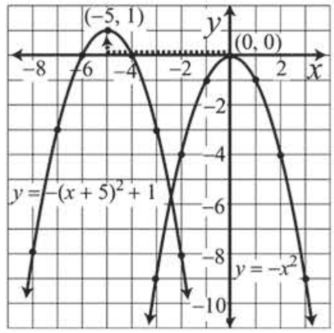
Vertex Form: $y = (x - h)^2 + k$

$$
\begin{aligned}
y &= x^2 + bx + c \\
&= (x-h)^2 + k \quad \text{where } h = -\frac{b}{2} \text{ and } k = -\frac{b^2 - 4c}{4}.
\end{aligned}
$$

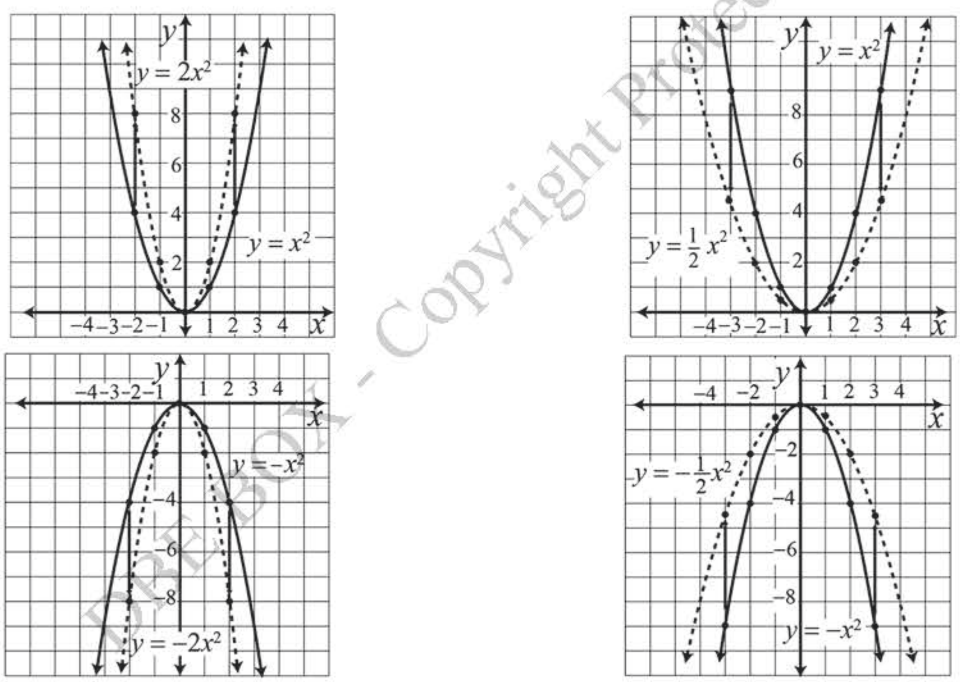


Vertex Form: $y = -(x - h)^2 + k$

$$
\begin{aligned}
y &= -x^2 + bx + c \\
&= -(x-h)^2 + k \quad \text{where } h = \frac{b}{2} \text{ and } k = \frac{b^2 + 4c}{4}.
\end{aligned}
$$

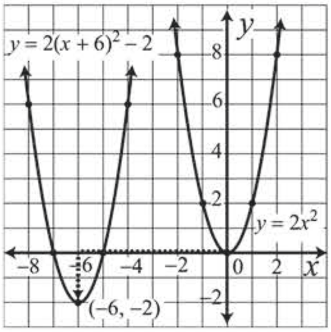


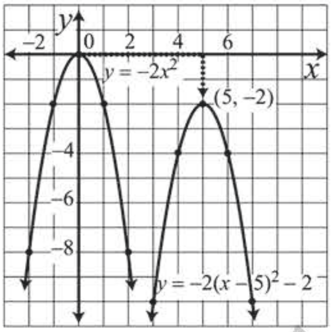
$y = ax^2$:



Vertex Form: $y = a(x-h)^2 + k$

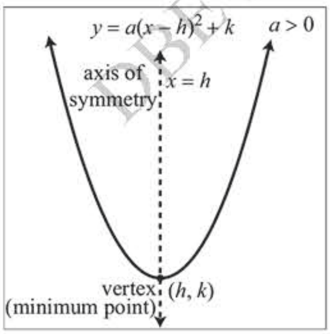
$$\begin{aligned} y &= ax^2 + bx + c \\ &= a(x-h)^2 + k \quad \text{where } h = -\frac{b}{2a} \text{ and } k = -\frac{b^2 - 4ac}{4a}. \end{aligned}$$

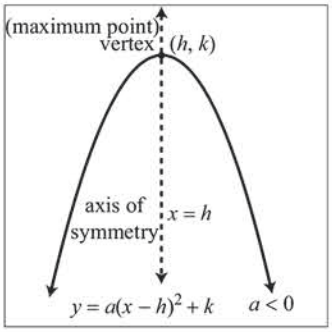




Feature of the graph: $y = ax^2 + bx + c = a(x - h)^2 + k$

- vertex: $(h, k) = (-\frac{b}{2a}, -\frac{b^2 - 4ac}{4a})$
- axis of symmetry: $x = -\frac{b}{2a}$
- y-intercept: $(0, c)$
- domain = the set $\mathbb{R}$ of all real numbers
- range = $\begin{cases} \{y \mid y \geq k\} & \text{when } a > 0; k \text{ is minimum} \\ \{y \mid y \leq k\} & \text{when } a < 0; k \text{ is maximum.} \end{cases}$





Discriminant: $b^2 - 4ac$.

When $b^2 - 4ac > 0$ the graph passes through the x-axis at two points.
When $b^2 - 4ac < 0$ the graph does not pass through the x-axis.
When $b^2 - 4ac = 0$ the graph meets the x-axis at exactly one point.

Factor Form: If $b^2 - 4ac \geq 0$, the quadratic function $y = ax^2 + bx + c$ can be written as in factor form $y = a(x - p)(x - q)$.

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- If $b^2 - 4ac > 0$, then the quadratic equation $ax^2 + bx + c = 0$ has two real solutions as $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$;
- if $b^2 - 4ac = 0$, then the quadratic equation has only one real solution (repeated solution) as $x = -\dfrac{b}{2a}$;
- if $b^2 - 4ac < 0$, then the quadratic equation has no real solution.

Quadratic Inequality: $y = ax^2 + bx + c$

Case 1. $\boxed{b^2 - 4ac < 0}$

$a > 0$		$a < 0$	
	solution set		solution set
$y < 0$	$\varnothing$	$y < 0$	$\mathbb{R}$
$y = 0$	$\varnothing$	$y = 0$	$\varnothing$
$y > 0$	$\mathbb{R}$	$y > 0$	$\varnothing$

Case 2. $\boxed{b^2 - 4ac = 0}$

$a > 0$		$a < 0$	
	solution set		solution set
$y < 0$	$\varnothing$	$y < 0$	$\mathbb{R} \setminus \{-\frac{b}{2a}\}$
$y = 0$	$\{-\frac{b}{2a}\}$	$y = 0$	$\{-\frac{b}{2a}\}$
$y > 0$	$\mathbb{R} \setminus \{-\frac{b}{2a}\}$	$y > 0$	$\varnothing$

Case 3. $\boxed{b^2 - 4ac > 0}$

If $b^2 - 4ac > 0$, then

$$y = ax^2 + bx + c = a(x-p)(x-q),$$

where p and q are x-intercepts $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ of the function. Let us assume that $p < q$.

$a > 0$		$a < 0$	
	solution set		solution set
$y < 0$	$\{x \mid p < x < q\}$	$y < 0$	$\{x \mid x < p \text{ or } x > q\}$
$y = 0$	$\{p, q\}$	$y = 0$	$\{p, q\}$
$y > 0$	$\{x \mid x < p \text{ or } x > q\}$	$y > 0$	$\{x \mid p < x < q\}$

Chapter 6

Absolute Value Functions

Total number of lesson periods (10), 1 period (45 minutes)

Learning Outcomes
Students will learn

- absolute value functions
- absolute value function equation $|x - p| = q$ and inequalities involving $|x - p|$

Skill Development
Student will learn

- graph of the function $y = |x - h| + k$
- graph of the function $y = -|x - h| + k$
- graph of the function $y = a|x|$
- graph of the function $y = a|x - h| + k$
- equation $|x - p| = q$
- inequalities involving $|x - p|$

Section 6.1 Graph of the function $y = |x - h| + k$

Number of lesson period (1)

Lesson Objective

- Draw graph of the function $y = |x - h| + k$.

Introduction

This is the first time students learn about absolute value functions. Definition of the absolute value of a real number is clearly stated and graphs of the functions like $y = |x|$, $y = -|x|$ and $y = |x - h| + k$ are presented. The graph of $y = |x - h| + k$ is only the translation of the graph of $y = |x|$.

Suggestion for teaching, practicing and evaluation

First draw the functions $y = |x|$ and $y = -|x|$. Then only emphasizes on $y = |x|$. On same graph paper draw the functions such as $y = |x|$, $y = |x| + 3$ and $y = |x|$. From these graphs students will learn vertical translations like up and down of the graphs of the functions. Then on same graph paper draw the graphs of the functions such as $y = |x|$, $y = |x - 2|$, and $y = |x + 3|$. From these graphs students will learn horizontal translations of the graphs of the functions. After that try to draw the graphs of the functions as in Example 1.

Section 6.2 Graph of the function $y = -|x - h| + k$

Number of lesson period (1)

Lesson Objective

- Draw graph of the functions $y = -|x - h| + k$.

Introduction

The graph of $y = -|x|$ is the reflection on the x-axis of the graph of $y = |x|$. The graph of $y = -|x - h| + k$ is only the translation of the graph of $y = -|x|$.

Suggestion for teaching, practicing and evaluation

On same graph paper draw the functions such as $y = -|x|$, $y = |x| + 3$ and $y = |x|$. From these graphs students will learn vertical translations like up and down of the graphs of the functions. Then on same graph paper draw the graphs of the functions such as $y = -|x|$, $y = -|x - 2|$, and $y = -|x + 3|$. From these graphs students will learn horizontal translations of the graphs of the functions. After that try to draw the graphs of the functions as in Example 2.

Section 6.3 Graph of the function $y = a|x|$

Number of lesson period (1)

Lesson Objective

- Draw graph of the functions $y = a|x|$.

Introduction

Drawing the quadratic function $y = a|x|$ when $a \neq 0$ is a necessary to know the absolute value functions.

Suggestion for teaching, practicing and evaluation

Draw at least two graphs for $a > 0$ and for $a < 0$ as shown in the text. If possible add some more drawing. Students need to see the vertical stretches of the graphs.

Section 6.4 Graph of the function $y = a|x - h| + k$

Number of lesson periods (2)

Lesson Objectives

- Draw graph of the functions $y = a|x - h| + k$ and features of the graph $y = a|x - h| + k$.

This section is the main part of the chapter. Drawing general absolute value function $y = a|x - h| + k$ is important to understand the nature of quadratic functions.

Suggestion for teaching, practicing and evaluation

May be needed some teaching aids like in section 5.1.

Section 6.5 Equation $|x - p| = q$

Number of lesson periods (2)

Lesson Objectives

- Find the solution of $|x - p| = q$, $|ax - p| = q$ and illustrate the equation $|x - p| = q$ on the number line.

Introduction
Distance between two points on the number line can be seen as an absolute value. Compare to solve the equation $|x-p| = q$ to find the points on the real line.

Suggestion for teaching
Equations like $|x - 5| = 3$ and $|x + 3| = 2$ are can be illustrated on the number line. But Equations like $|2x - 5| = 4$ and $|-2x-4| = 3$ are just for solving.

Section 6.6 Inequalities Involving $|x-p|$

Number of lesson periods (3)

Lesson Objectives

- Solve the inequalities involving $|x - p|$, $|ax - p|$ and illustrate the inequality involving $|x-p|$ on the number line.

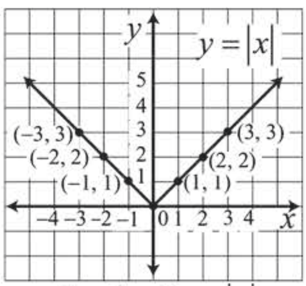
Introduction
Every inequality involving $|x-p|$ can be solved. This section explains a method for solving these inequality.

Suggestion for teaching, practicing and evaluation
Four cases of inequality have to teach. After that there is no difficulty for solving these inequality.

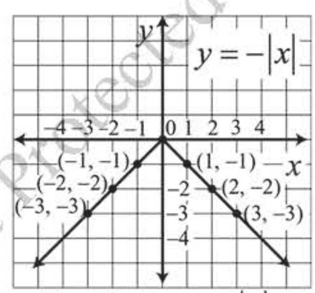
SUMMARY

Absolute value: For a real number x, the **absolute value** or **modulus of** x, which is written as $|x|$, is defined as follow:

$$\boxed{|x| = \begin{cases} x, & \text{when } x \geq 0, \\ -x, & \text{when } x < 0. \end{cases}}.$$

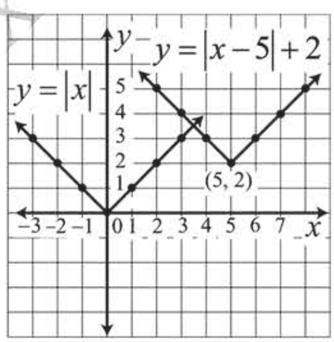


Graph of $y = |x|$

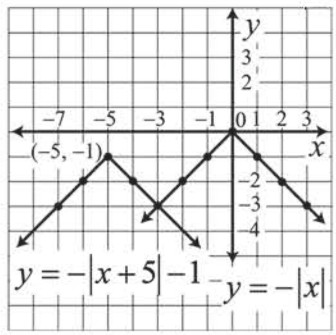


Graph of $y = -|x|$

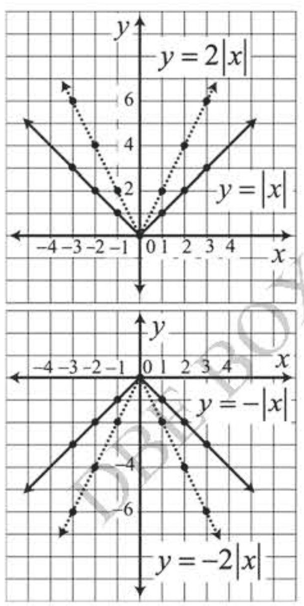
Graph of the function: $y = |x - h| + k$

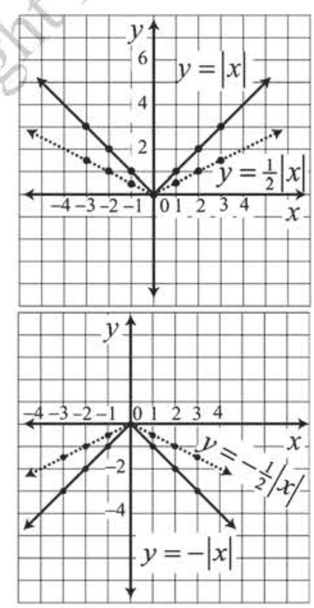


Graph of the function: $y = -|x - h| + k$

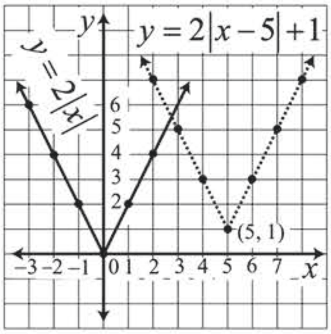


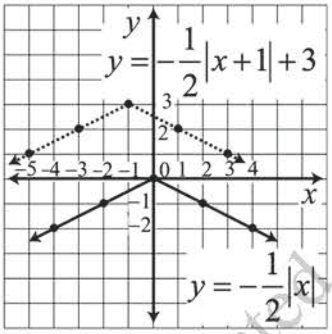
Graph of the function: $y = a|x|$





Graph of the function: $y = a|x-h|+k$





Feature of the graph: $y = a|x-h|+k$

Features of the graph $y = a|x-h|+k$

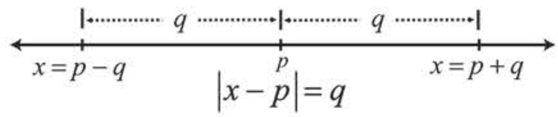
- vertex: (h, k)
- axis of symmetry: $x = h$
- domain = the set $\mathbb{R}$ of all real numbers
- range $= \begin{cases} \{y \mid y \geq k\} & \text{when } a > 0 \\ \{y \mid y \leq k\} & \text{when } a < 0 \end{cases}$
- $\begin{cases} \text{when } a > 0, & k \text{ is the minimum value of } y \\ \text{when } a < 0, & k \text{ is the maximum value of } y \end{cases}$

Equation: $|x-p| = q$

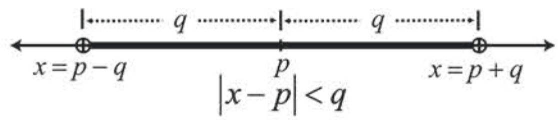
- When $q < 0$, $|x-p| = q$ has no solution.
- When $q = 0$, $|x-p| = 0$ has only one solution p.
- When $q > 0$, the equation $|x-p| = q$ can be seen as

$$x - p = q \text{ or } x - p = -q.$$

So the solutions are $x = p + q$ and $x = p - q$.

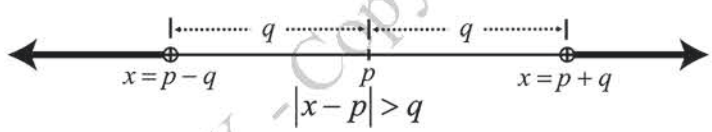


$|x-p|<q$: The solution set of $|x-p|<q$ when $q>0$ is $\{x|\, p-q<x<p+q\}$.



$x-p|>q$: The solution set of $|x-p|>q$, when $q \geq 0$ is

$$\{x|\, x<p-q \text{ or } x>p+q\}$$



Chapter 7

Probability

Total number of lesson periods (10), 1 period (45 minutes).

Learning Outcomes

The learners are able to

- be familiar with the terminology used in probability theory.
- understand probability in theoretical point of view, on the assumption that all outcomes in the experiments are equally likely.
- evaluate the probabilities of individual events, complements of events and combined events, analyzing their properties with respect to mutually exclusiveness and independence.
- estimate the frequency of an event that can occur in a finite number of trials.

Skill Development

Students will be able to

- understand the basic concept of probability.

- enumerate the possible outcomes mentally or by using a special techniques such as drawing tree diagrams and constructing tables, and distinguish the outcomes that are favorable to the events considered.
- calculate the probabilities of single events as well as combined events, and furthermore enhance logical consideration.
- guess frequencies of an events, occurring in a number of trials, depending on their probabilities.

Section 7.1 Calculating Probability

Number of lesson periods (4)

Lesson Objectives

Basic terminology and concept of probability theory will have to be introduced. Probability, in fact, can be defined in a slightly different ways – theoretically, experimentally or subjectively. In this lesson we emphasize on theoretical point of view, so all outcomes in the experiments are assumed to be equally likely.

Introduction

Teachers will have to discuss their students about chance processes and likelihood of particular outcomes. This kind of task can be systematically performed using probability theory. In 16th century, a professional gambler named Chevalier de Méré contacted a mathematician Blaise Pascal (1623-1662) to find out his problem on a gambling game. Pascal became interested and began studying probability theory. Together with Fermat (1601-1665), he formulated the beginnings of probability theory.

Suggestion for teaching, practicing and evaluation

Motivation can be made by tossing a coin or rolling a die, the materials are easily available; otherwise some relevant gaming applications can be downloaded from the the internet to smart phones and let them play to some extent. Then discuss, depending on the context, on the techniques of finding possible outcomes, including the terms: experiments, outcomes and events. After that the definition of probability should be explained, noting that the outcomes are assumed to be equally likely for this definition.

There are six examples in this section, after completing them, seven problems in Exercise 7.1 should be practiced. Tests can be assigned by generating similar questions to evaluate the students' skills.

Section 7.2 Probabilities of Combined Events

Number of lesson periods (4)

Lesson Objectives

In this section, probabilities of complements of events and combined events will have to be taught, describing their properties with respect to mutually exclusiveness and independence.

Introduction

This section about the evaluation of the combinations of outcomes. Motivate the students by discussing the case of choosing a marble from a bag and rolling a die, and the events stated in the text. It is possible for the teacher to discuss other cases which the students may supposed to be familiar with.

Suggestion for teaching, practicing and evaluation

Teachers must be careful in teaching topics in this section, which contains the study of probabilities of the events in the combinations in comparison with those of the whole combinations.

Practicing can be made by thorough discussion on the examples and exercises; based on which the additional questions can be appropriately transformed or generated for the test for evaluation.

7.3 Calculation of Expected Frequency

Number of lesson periods (2)

Lesson Objectives

The section concerns with expected frequency of an event which may suppose to occur in a number of trials. The frequencies are estimated depending on the probability of the events concerned.

Introduction

The topic in this section is very simple to be introduced. Motivation can be made by successively tossing a coin about ten times, students will have to asked how many times a head may occur. Their answers are in fact expected frequency and the the actual frequency may be different from their answers. The latter is also called the experimental frequency in contract to the expected frequency.

Suggestion for teaching, practicing and evaluation

The topic in this is simple. For practicing, students need to complete examples and exercises presented in the text. Tests for evaluation can be made in a similar fashion done for previous sections.

SUMMARY

1. Calculating Probability:

Let S be a finite sample space for an experiment such that all outcomes are *equally likely.* Then the *probability of an event* A, denoted by $\mathrm{P}(A)$, is defined by

$$\mathrm{P}(A) = \frac{\text{number of outcomes in the event } A}{\text{number of outcomes in the sample space } S}$$

2. Probabilities of Combined Events:

Multiplication Rule :

If A and B *independent* events in a sample space *if and only if*

$$\mathrm{P}(A \text{ and } B) = \mathrm{P}\,(A) \times \mathrm{P}\,(B).$$

Addition Rule 1 :

For any two events A and B in a sample space, then

$$\mathrm{P}(A \text{ or } B) = \mathrm{P}\,(A) + \mathrm{P}\,(B) - \mathrm{P}(A \text{ and } B).$$

Additioin Rule 2 :

If A and B are *mutually exclusive* events in a sample space, then

$$\mathrm{P}(A \text{ or } B) = \mathrm{P}\,(A) + \mathrm{P}\,(B)\,.$$

3. Calculation of Expected Frequency:

$$\begin{matrix} \textit{Expected frequency} \\ \textit{of an event} \end{matrix} = \begin{matrix} \textit{Probability} \\ \textit{of the event} \end{matrix} \times \begin{matrix} \textit{Number} \\ \textit{of trials} \end{matrix}$$

Chapter 8

Similarity

Total number of lesson periods (20), 1 period (45 minutes)

Learning Outcomes
It is expected that students will be able to

- recognize and classify such plane figures as triangles, parallelogram, rectangles, squares;
- determine whether two given figures are similar and to use similarity to find missing lengths;
- know how to do such basic constructions as bisecting an angle, constructing a perpendicular to a line, and copying a triangle;
- understand such relationship as those dealing with the angles formed by parallel lines as well as those concerning similarity of triangles;(AAA, AA, SAS, SSS)
- understand the nature of proofs.
- increase the knowledge of facts, and to select the more important as standard theorems, which can be used for this deductive process.
- develop both geometric and algebraic modes of reasoning.

Skill Development

Develop an appreciation of deductive reasoning, and a power to use it.
Develop a better understanding of geometric concepts.
Develop to define similarity of shapes based on similar triangles or intuitive knowledge.

Section 8.1 The Ideas of Similarities and Similar Triangles
Section 8.2 Basic Proportionality Theorem

Number of lesson periods (5)

Learning Objectives

At the end of the lesson the students will be able to solve the geometric problems by using the basic proportionality theorem. The students must know that similar triangles have equal corresponding angles and corresponding sides in the same ratio.

Introduction

Depending on the problems, the solutions such as marked lengths, proportionality of sides and similarity of triangles can be solved by using theorem.

Suggestion for teaching, practicing and evaluation

The teacher must ask students the question such that when will the two triangles be similar? The teacher would explain that triangles whose corresponding angles are equal and whose corresponding sides are proportional are said to be similar.

The teacher would explain the Basic Proportionality Theorem. The students must know if three parallel lines are intersected by two transversals, then the lines divide the transversals proportionally.

The teacher has to write Theorem(1)(The Basic Proportionality Theorem), Corollary 1 and 2 on the blackboard and also explain these theorem and corollaries with the knowledge of geometry to the pupils.

The teacher would explain that triangles whose corresponding angles are equal and whose corresponding sides are proportional are said to be similar. The students must know if three parallel lines are intersected by two transversals, then the lines divide the transversals proportionally.

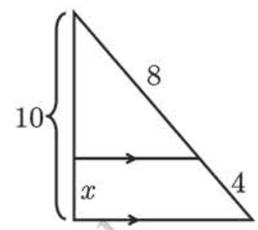
The teacher has to give the following activity to find the value of x in the figure.

A student wrote the proportion $\frac{x}{10} = \frac{4}{8}$.

(1) Is this correct?

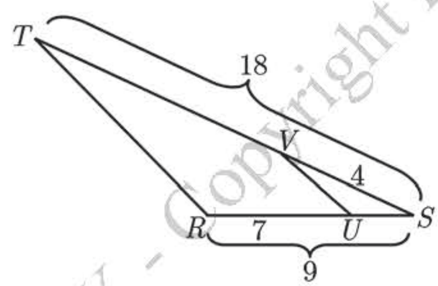
(2) Another student wrote the proportion $\frac{x}{x-4} = \frac{4}{8}$.

Is this correct?



(3) Write a simpler proportion that will given the correct answer.

Exercise: If the segments in the figure have the legths indicated.
Is $UV \parallel RT$? Jusitfy your answer.



Section 8.3 Basic Theorems on Similar Triangles

Number of lesson periods (5)

Lesson Objective

The students will be able to name a similarity correspondence for the triangles.

Introduction

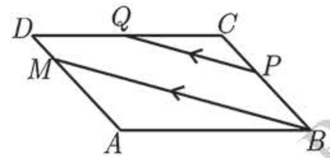
By using basic theorems on similar triangles, the pairs of triangles which are similar or not are determined, and also calculate the unknown lengths depending on similarity triangles.

Suggestion for teaching, practicing and evaluation

The teacher must ask students the question such that how will you prove to be similar of triangles? The teacher has to write Theorem 2, Corollary 3, 4, Theorem3, 4 on the blackboard and also must explain these theorems and corollaries to the pupils.

The teacher has to explain how to the corresponding angles are represented in this expression and the pupils have made a correspondence between the similar trangles which also indicates the corresponding angles. The following activity is given the pupils to prove the similarity of triangles. The teacher has to draw the figure:

Exercise: $ABCD$ is a parallelogram and $PQ \parallel MB$. Prove that $\triangle ABM \sim \triangle CQP$.



Section 8.4 The Angle Bisector Theorem
Section 8.5 The Pythagoras Theorem

Number of lesson periods (5)

Lesson Objectives

Learn the notion of dividing a segment internally or externally in a given ratio. Understand the relationship between the three sides of a right triangle.

Introduction

For a given segment there are usually two points which divide the segment in the ratio. One it an internal point , the other is an external point. It is extended as theorem in which the bisector of an interior(exterior) angle of a triangle divides the opposite side internally (externally) into a ratio equal to the ratio of the other two sides of the triangle.
Pythagoras was a Greek mathematician whose theorem is the most famous theorem in geometry used to calculate the length of the sides in a right triangle. The converse of the Pythagoras Theorem provides a way of showing whether or not a triangle is a right triangle.

Suggestion for teaching, practicing and evaluation

The teacher must ask students the question such that what is the angle bisector theorem. How the angle bisector in a triangle bisects the opposite side? What is the Pythagoras Theorem? The teacher has to write Theorem 5, 6, 7 and explain these examples. And then the teacher must prove to Pythagoras Theorem(7).

Section 8.6 Special Right Triangles

Number of lesson periods (5)

Lesson Objectives

The student will be able to know two special types of right triangle and determine the relationship between the lengths of their legs and hypotenuse.

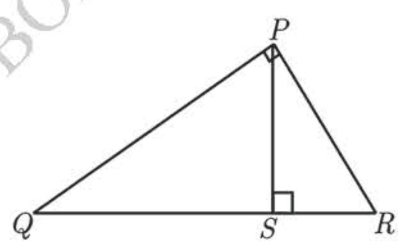
Introduction

There are two special types of right triangles one is isosceles right triangle and the other is the right triangle with acute angles of measures 30° and 60°. In a 45°-45° right triangle, the length of hypotenuse is equal to the length of each leg times 2. In a 30°-60° right triangle, the leg opposite to 30° angle is one-half the length of hypotenuse.

Suggestion for teaching, practicing and evaluation

The teacher write the Theorem 8, 9 and explain the students the relation of their measure of lengths of these triangle.

Exercise: The triangle PQR, $\angle QPR = 90°$, $PS \perp QR$. Prove that $\dfrac{1}{PS^2} = \dfrac{1}{PQ^2} + \dfrac{1}{PR^2}$.



SUMMARY

1. **The Means-extremes Product Property:** The product of the means equals the product of the extremes. If $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$, where a, d are extremes and b, c are means. If $\frac{a}{b} = \frac{b}{c}$, then b is called the **geometric mean** of a and c.

2. **Invertendo Property:** In a proportion, the ratios may be inverted. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$.

3. **Alternando Property:** In a proportion, the means (or extremes) may be interchanged. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$ (or $\frac{d}{b} = \frac{c}{a}$).

4. **Componendo Property:** If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$.

5. **Dividendo Property:** If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$.

6. **Componendo and Dividendo Property:** If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

7. In any trianlge
   (a) a line parallel to one side and intersecting the other two sides divides them proportionally. (BPT)
   (b) a ray that bisects an angle divides the oppsite side into segments whose lengths are proportional to the lengths of the two sides.(ABT)

8. In similar triangles
   (a) corresponding angles have equal measures.
   (b) the lengths of corresponding sides are in proportion.
   (c) the ratio of the lengths of two corresponding altitudes is equal to the ratio of the lengths of two corresponding sides.

9. Two trianlges are similar

(a) if two angles of one triangle are congruent two angles of the other triangle.(AA)

(b) if the lengths of two sides of a triangle are proportional to the lengths of two sides of another triangle and the included angles are congruent.(SAS)

(c) if the lengths of their corresponding sides are proportional.(SSS)

10. In any right trianlge

(a) the altitude to its hypotenuse forms two right triangles that are similar to each other and to the given triangle.

(b) the altitude drawn to the hypotenuse is geometric mean of, or (the mean proportional between) the segments of the hypotenuse.

Chapter 9

Circles

Total number of lesson periods (10), 1 period (45 minutes)

Learning outcomes

It is expected that students will be able to

- know properties of angles in a circle
- know properties of chords

Skill Development

Students will be able to

- analyses problems and to know constructive proofs and formal proofs
- develop reasoning under given conditions
- increase the knowledge in problem solving strategies

Section 9.1 Angles in a Circle

Number of lesson periods (4)

Lesson Objectives

Students should be able to know the properties of angles in a circle and prove related problems.

Introduction

A **central angle** is an angle subtended by an arc (or chord) of a circle at the centre. An **inscribed angle** is an angle subtended by an arc (or chord) of a circle at a point on the other arc. A quadrilateral whose vertices lie on a circle is called a **cyclic quadrilateral**.

The relations between angles in the same circle or in congruent circles are:

- central angles and inscribed angles,
- inscribed angles subtended by the same arc(chord),
- inscribed angle subtended by a diameter,
- opposite angles of a cyclic quadrilateral,
- central angles and corresponding arcs,
- inscribed angles and corresponding arcs.

Suggestion for teaching, practicing and evaluation

Students should know the words **central angle** and **inscribed angle**. After that teachers should explain the following theorems:

- The central angle is twice the inscribed angle subtended by the same arc.
- Inscribed angles subtended by the same arc are equal.
- An inscribed angle subtended by a diameter is a right angle.

After explaining the word **Cyclic Quadrilateral**, teachers should explain the following theorems:

- Opposite angles of a cyclic quadrilateral are supplementary.
- The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle of the quadrilateral.

The relations between central angles and arcs are stated in the following theorem.

In the same circle or in congruent circles,

(i) equal arcs subtend equal central angles,

(ii) arcs subtending equal central angles are equal.

Teachers should explain the following statement related with the above theorems.

- In the same circle or in congruent circles, two inscribed angles are equal if and only if the corresponding arcs are equal.

Firstly, do practice with No. 1 of Exercise 9.1 by asking the reasons to find unknown values. Make students to read the questions and to draw the figure correctly. Make clearly given facts and what is to prove. Please analyse theorems or corollaries to solve. Some of the problems are needed auxiliary drawings.

9.2 Properties of Chords

Number of lesson periods (6)

Lesson Objectives
Students should know relations among chords, central angles, arcs and distances of chords from the centre. Moreover students should know product properties of chords and secants.

Introduction
In Theorem 5, relation between chords and central are stated. Theorem 6 to Theorem 9 are symmetric properties of chords. Comparison of chords are stated in Theorem 10. Theorem 11 is about product property of two chords intersecting in a circle. Theorem 10, the last theorem, is about product property of chord and secant.

Suggestion for teaching, practicing and evaluation
Teachers should explain the following statements.
In the same circle or in congruent circles,

(i) equal chords subtend equal central angles,

(ii) equal central angles cut off equal chords.

- In the same circle or in congruent circles, two central angles are equal if and only if two corresponding minor arcs are equal if and only if two corresponding chords are equal.

The following are symmetrical properties of chords.

- If a line passing through the centre of a circle bisects a chord of the circle, then the line is perpendicular to the chord.
- If a line passing through the centre of a circle is perpendicular to a chord of the circle, then the line bisects the chord.
- The perpendicular bisector of a chord of a circle passes through the centre.
- In the same circle or in congruent circles, chords are equal if and only if they are equidistant from the centre of the circle.

The following theorem is a comparison of chords.

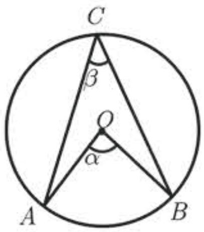
- Of any two chords of a circle, the greater chord is nearer to the centre, and conversely, the chord nearer to the centre is larger.

The following theorem are product properties of chords and secants.

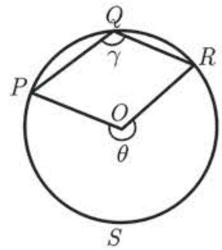
- If two chords of a circle intersect in the circle, the product of the lengths of segments of one chord is equal to the product of the lengths of segments of the other chord.
- If two secants are drawn to a circle from an external point, the product of the lengths of one secant and its external segment is equal to the product of the lengths of the other secant and its external segment.

Firstly, do practice with No. 1 through No. 3 of Exercise 9.2 by asking the reasons to find unknown values. No. 4 through No. 6 are arranged in sequence. Make students to read the questions and to draw the figure correctly. Make clearly given facts and what is to prove. Please analyse theorems or corollaries to solve. Some of the problems are needed auxiliary drawings.

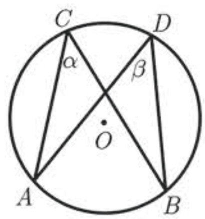
SUMMARY



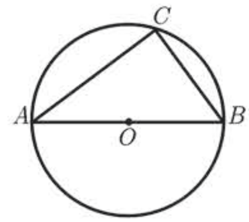
$\alpha = 2\beta$



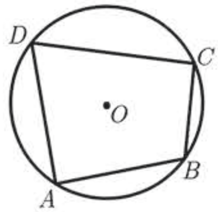
$\theta = 2\gamma$



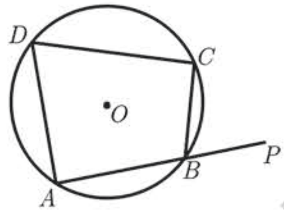
$\alpha = \beta$



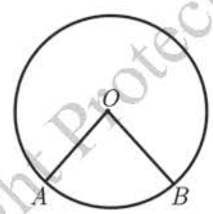
$\angle ACB = 90^\circ$

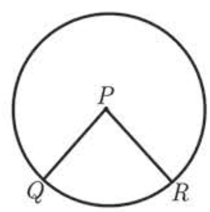


$\angle A + \angle C = 180^\circ$



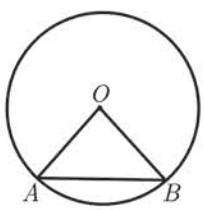
$\angle CBP = \angle D$

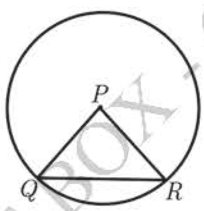




$\odot O$ and $\odot P$ are congruent

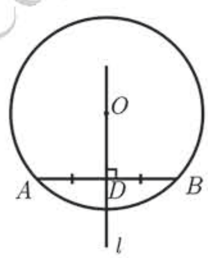
$\angle AOB = \angle QPR \Leftrightarrow \text{arc } AB = \text{arc } QR$





$\odot O$ and $\odot P$ are congruent

$\angle AOB = \angle QPR \Leftrightarrow AB = QR$

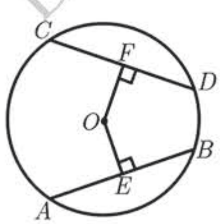


If any two of the following conditions are satisfied, the other is also satisfied

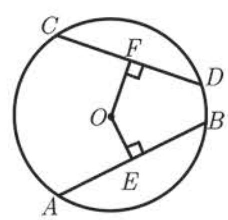
(i) l passes through centre O

(ii) l bisects AB

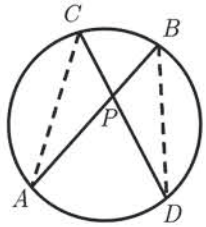
(iii) $l \perp AB$

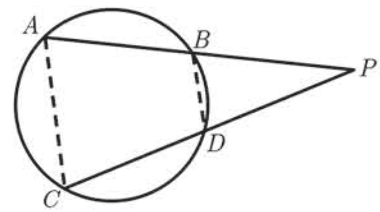


$AB = CD \Leftrightarrow OE = OF$



$AB > CD \Leftrightarrow OE < OF$





$$PA \cdot PB = PC \cdot PD$$

Chapter 10

Trigonometry

Total number of lesson periods (20), 1 period (45 minutes)

Learning Outcomes
Students will be able to

- consider the angles in each of the quadrants of the plane
- know the degree measure and radian measure
- find the arc length and the area of a sector
- know the basic trigonometric ratios
- know the relations between (6) ratios and derived the square formulas and use an expression
- find the value of trigonometric ratios by using special angles
- solve the problems related with angle of elevations and angle of depressions

Skill Development
Students will be able to

- use radian measure to help learning science subjects

- solve the rest of the angles, the other sides skillfully
- use basic formulae and square formulae to calculate in proving identities
- estimated the buildings and homes the low or high rise by learning the angle of elevation and angle of depression

Section 10.1 Angles

Section 10.2 The Relation between Degree and Radian Measure

Number of lesson periods (3)

Lesson Objectives
Learn how to use the angle measure. Find out the relation between degree and radian measures.

Introduction
An angle is determined by rotating a ray about its endpoint from an initial position to terminal position. Angles measured from the X-axis (i.e., OX) in an anticlockwise direction are positive angles. Angles measured from the X-axis (i.e., OX) in a clockwise direction are negative angles. Consider a line OP which is free to rotate in the XY-plane. O is taken to be the origin about which a line OP rotates.

Two kinds of units commonly used for measuring angles are radian measure and degree measure. The radian measure is employed almost exclusively in advanced mathematics and in many branches of science. In this chapter first we introduce the concept of radian and study the relation between degrees and radians.

Suggestion for teaching, practicing and evaluation
Teach how to properly use angle measure. Be sure to teach the relation between degree and radian measures. One complete revolution is an angle of 360°. The range of values of between 0° and 360°. Since the circumference of a circle is equal to $2\pi r$, it subtends a central angle of 2π radians. That is, there are 2π radians in a complete rotation of 360°.

Therefore 2π radians $= 360°$ and hence π radians $= 180°$ which is a fundamental relation between radians and degrees.

We have 1 radian $= \dfrac{180}{\pi}$ degrees $\approx 57°19'$ and $1° = \dfrac{\pi}{180}$ radians ≈ 0.01764 radians.

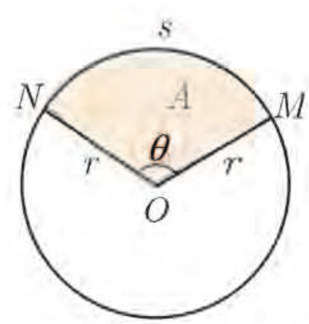
Section 10.3 Arc Length and Area of a Sector of a Circle

Number of lesson periods (3)

Lesson Objectives
Find out the connection between arc length, central angle and radius. To find the area of sector of a circle.

Introduction
For a sector of a circle enclosed by two radii that make an angle of θ radians at the centre, the arc length s is given by $s = r\theta$ and the area of the sector of a circle, A is given by $A = \frac{1}{2}r^2\theta$ where r is the radius of the circle.

Suggestion for teaching, practicing and evaluation
The teacher derive the following formulae.



Let the arc MN subtend an angle of magnitude θ radians at the centre of a circle of radius r as shown in figure. Clearly the length s of arc MN is proportional to the angle θ and we have

$$
\begin{aligned}
\frac{\text{length of arc } MN}{\text{length of the circumference}} &= \frac{\text{angle subtended by arc } MN}{\text{angle subtended by circumference}} \\
\frac{s}{2\pi r} &= \frac{\theta}{2\pi} \\
\text{(or) } \theta(\text{in radians}) &= \frac{s}{r} \\
\text{(or) } \qquad s &= r\theta
\end{aligned}
$$

Furthermore, as the area of the sector MON (the shaded region shown in the figure) is also, proportional to the angle θ, we have

$$
\begin{aligned}
\frac{\text{area of sector } MON}{\text{area of circle}} &= \frac{\theta}{2\pi} \\
\frac{A}{\pi r^2} &= \frac{\theta}{2\pi} \\
A &= \frac{1}{2} r^2 \theta
\end{aligned}
$$

Section 10.4. Six Trigonometric Ratios

Number of lesson periods (3)

Lesson Objectives
To find the six trigonometric ratios from the right triangle. To determine the opposite side, adjacent side and hypotenuse.

Introduction
If the lengths of two sides of a right triangle are given, the length of the third side can be calculated by using the Pythagoras theorem. So, if the value of one of the six trigonometric ratios is given, then the values of the other five ratios can be found from the right triangle.

Suggestion for teaching, practicing and evaluation
Let the triangle ACB be a right triangle.

Denote the lengths of the segments BC, AC, AB by the letters a, b, c respectively.

We say that BC is the side which is opposite to angle A; AC is the side which is adjacent to angle A; AB is the hypotenuse. With reference to the angle A the following definitions are employed. Then find the six ratios as sine, cosine, tangent, cotangent, secant and cosecant.

Section 10.5 Relations between the Trigonometric Ratios

Number of lesson periods (3)

Lesson Objective
From the six ratios to find the relations between the trigonometric ratios.

Introduction
Let ABC be a right triangle with $\angle C = 90°$.

Denote the lengths of the segments BC, AC, AB by the letters a, b, c respectively.

Suggestion for teaching, practicing and evaluation
Three types of identities are considered in this text.

(i) The reciprocal relations
Derive the relations between $\sin A$ and $\csc A$, $\cos A$ and $\sec A$, $\tan A$ and $\cot A$.

(ii) The Pythagorean identities
Also derive $\tan A = \dfrac{\sin A}{\cos A}$, $\cot A = \dfrac{\cos A}{\sin A}$, $\sin^2 A + \cos^2 A = 1$, $1 + \tan^2 A = \sec^2 A$ and $1 + \cot^2 A = \csc^2 A$.

(iii) The ratios of two complementary angles
You need to show that $\sin(90° - \alpha) = \cos\alpha$, $\cos(90° - \alpha) = \sin\alpha$, $\tan(90° - \alpha) = \cot\alpha$, $\cot(90° - \alpha) = \tan\alpha$, $\sec(90° - \alpha) = \csc\alpha$, $\csc(90° - \alpha) = \sec\alpha$.

Section 10.6 Value of the Trigonometric Ratios for Some Special Angles

Number of lesson periods (3)

Lesson Objective
To know that the value of the trigonometric ratios for some special angles.

Introduction
From 45°-45° right triangle, find the trigonometric ratios for an angle of 45°
From 30°-60° right triangle, find the trigonometric ratios for an angle of 30° and 60°.

Suggestion for teaching, practicing and evaluation
Write the six trigonometric ratios for an angle of $45°, 30°$ and $60°$.

Section 10.7 Solution of Right Triangles

Number of lesson periods (3)

Lesson Objective
To solve the right triangle.

Introduction
Every triangle has six parts, namely, three sides and three angles. In the solution of right triangles there are really only two cases to be considered:

Case(1) To solve a right-angled triangle when two sides are given.

Case(2) To solve a right-angled triangle when one side and one acute angle are given.

Suggestion for teaching, practicing and evaluation

Case(1) Let $\triangle ABC$ be a right-angled triangle with $\angle B = 90°$.
Suppose that any two sides are given.
Then the third side may be found from the equation $b^2 = a^2 + c^2$.
Also $\sin A = \frac{a}{b}$, and $\angle C = 90° - \angle A$; whence $\angle A$ and $\angle C$ may be obtained.

Case(2) Let $\triangle ABC$ be a right-angled triangle with $\angle B = 90°$.
Suppose that one side c and one acute angle A are given.
Then $\angle C = 90° - \angle A$, $\frac{b}{c} = \sec A$, and $\frac{a}{c} = \tan A$; whence $\angle C, b$ and a may be obtained.

Section 10.8 Angle of Elevation and Angle of Depression

Number of lesson periods (2)

Lesson Objective

To know that the angle of elevation and the angle of depression.

Introduction

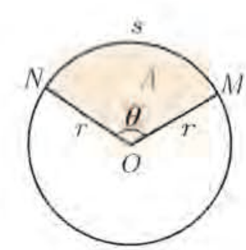
Suppose we are viewing an object. The line of sight or line of vision is a straight line from or eye to the object we are viewing. If the object is above the horizontal from the eye, we have to move our head lift up to view the object. In the process, our eye moves through an angle. This angle is called the **angle of elevation**. If the object is below the horizontal from the eye, we have to move our head downwards to view the object. In the process, our eye moves through an angle. This angle is called the **angle of depression**.

Suggestion for teaching, practicing and evaluation

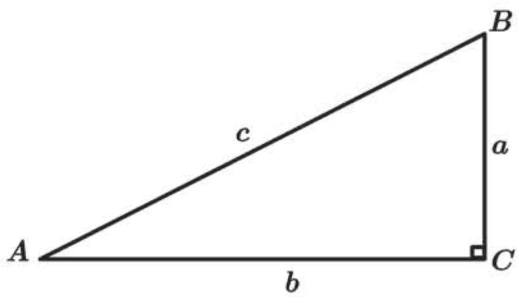
Let us assume that we are on a boat on the ocean and want to find the height (altitude) of a mountain in the distance. First, at point A in the ocean we measure the angle of elevation α by sighting to the top of the mountain. Then we move our boat a distance of d km straight toward the mountain to point B. At point B we again measure the angle of elevation β. We can determine the height h of the mountain by making use of the trigonometric ratios. We should convey to the students an understanding that this kind of calculation is possible because of the properties of right triangles and the principle of ratio. Try to solve the counter examples and exercises.

SUMMARY

1. π radians $= 180°$.
2. For a sector of a circle enclosed by two radii that make an angle of θ radians at the centre, the arc length s is given by $s = r\theta$ and the area of the sector of a circle, A is given by $A = \dfrac{1}{2}r^2\theta$ where r is the radius of the circle.



3. Six Trigonometric Ratios



Ratio	Abbreviation	Definition	For right $\triangle ABC$
sine of $\angle A$	$\sin A$	$\dfrac{\text{oppsite side of } \angle A}{\text{hypotenuse}}$	$\dfrac{a}{c}$
cosine of $\angle A$	$\cos A$	$\dfrac{\text{adjacent side of } \angle A}{\text{hypotenuse}}$	$\dfrac{b}{c}$
tangent of $\angle A$	$\tan A$	$\dfrac{\text{opposite side of } \angle A}{\text{adjacent side of } \angle A}$	$\dfrac{a}{b}$
cotangent of $\angle A$	$\cot A$	$\dfrac{\text{adjacent side of } \angle A}{\text{opposite side of } \angle A}$	$\dfrac{b}{a}$
secant of $\angle A$	$\sec A$	$\dfrac{\text{hypotenuse}}{\text{adjacent side of } \angle A}$	$\dfrac{c}{b}$
cosecant of $\angle A$	$\csc A$	$\dfrac{\text{hypotenuse}}{\text{opposite side of } \angle A}$	$\dfrac{c}{a}$

4. Value of the Trigonometric Ratios for Some Special Angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$